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GEOMETRY I.

MANUAL FOR TEACHERS

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Foreword

PRESENT COURSE in Geometry has been developed ccordance with the new spirit in the curriculum and adion of the secondary schools.

text presupposes that the aim of education in a demoy is training for citizenship and as a recent writer puts it Such an aim will centre in the capacity for independent thought." The success of a given school according to such an aim is measured largely by its success in developing robust and resourceful thinking in its students. The success of the teacher with the new course in geometry can be estimated in the same way.

It is expected that the first six chapters of the text, largely review materials, will be taken up during the first six weeks. These chapters may also prove useful to the teacher of Grade IX mathematics and to the teacher of general mathematics. The remainder of the book will require some selection on the part of the teacher. It is hoped however, that the matters of main concern in these chapters will receive adequate attention.

Much of the flexibility of the text resides in the abundance of exercises. There are many more than can be done in a given year, and the slower pupil may well be spared the harder theory and exercises. The faster the pupil the more he should be encouraged to do the proof work in contrast to drawing and calculations.

The abundance of exercises also furnishes the teacher with plentiful test material.

References to algebra and the use of co-ordinates occur mainly in the exercises. These may be used with discretion; any tie-up to the students' knowledge of algebra is of course highly desirable.

It is hoped that some field work may be found possible in certain schools. There is plenty of scope for experimentation in this direction.

No attempt has been made in the text to collect or exhibit illustrations of geometry in design and in nature. If desired, the pupil can be on the look out for such material and can keep a note book for his findings. Each item should include some simple analysis of the figure or concept it is supposed to illustrate.

While the text has many figures, many more could have been added, and it is hoped that the teacher and the student will supply this deficiency where needed.

The notes on the successive chapters which follow take little or no account of the time which the teacher has at his or her disposal.

CHAPTER I

Angles, Parallels and Perpendiculars

The chapter should be taught informally. The quiz of Section **1** and the exercises of Section **4** will serve to locate student difficulties.

The formal summary at the end of the chapter may also be used for quizzing purposes. Are all the terms understood? What are the given things in the propositions? What are the required things?

Sections **2** and **3** are concerned with proving propositions. The aim should be to have the student begin to appreciate that the proof of a proposition is made in terms of other propositions some of which are themselves unproved, some proved.

The method of elimination in Section **3** and the material of Section **5** are new for the student.

Detail by Sections

1. There need be no great preparation for the quiz. The quiz, as on the radio, may be conducted by a leader, with small rival teams. The results should indicate pupil difficulties with angle concepts.

2. The layout may well be made in the school yard with convenient dimensions. A good tape will be needed; the rest of the equipment can be made by the class. For a transit all that is needed is a board calibrated for angle with a movable sight arm swinging over it. The board may be mounted on a camera tripod. A plumb line should be used. Such layout work will increase the pupils' respect for angle theory. As Bill discovered, it is difficult to recognize parallelism in large scale work.

3. The method of elimination was used by the Greek geometers especially in the form:

"If A is not greater than B and if A is not less than B then A is equal to B."

It is frequently used in the proof of converses; the converse is assumed to be false and a contradiction is then obtained (*reductio ad absurdum*).

4. Angles should be labelled as in these exercises, with small letters, whenever this can be conveniently done, or numbers 1, 2, 3, etc., may be used. The small letter is in keeping with the pupils' algebraic experience. The three-letter method of referring to angles is often unavoidable, however. R, S, T are better than the traditional A, B, C for class-room use, but the student should be able to use different sets of letters. This is arranged for in the exercises. The student should learn to write large and small letters in clear distinction. Certain small letters may be underlined with advantage.

In such figures as fig. 5, in both oral and written work, it is often convenient to colour the one pair of lines yellow and the other green and to give references thus:

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Angle q = cor. angle f, green lines parallel.

There is less distraction in naming the colour than in searching for the letters that name the lines.

5. There is a good deal of informal space-thinking in the text. In this paragraph, the pupil will see familiar notions in a new setting, as for example parallelism.

Chapter Summary

It was decided after some thought to follow the usage of *Siddons* and *Hughes* in marking with (‡) any theorem to be assumed throughout the course, and with (†) any theorem which may well be assumed, but which may be proved in terms of the propositions which precede it.

Our hesitation in this matter was concern lest the student be led to think "once an assumption always an assumption" or "once a theorem always a theorem."

The scheme referred to above has the disadvantage of playing up the word *theorem.* The teacher may well stress the word *proposition,* as something which may be agreed to without proof (assumption), may be rejected as inconsistent with earlier assumptions and theorems, or may be proved (theorem), i.e., may be shown to be consistent with the earlier propositions.

The student who wishes to try to prove assumed propositions should not be discouraged from doing so. It is hoped that as he proceeds through the text he will realize that

(*a*) An assumption may (1) turn out later to be inconsistent with other propositions (in which case something has to go) ; (2) be capable of proof in terms of other assumptions and theorems; (3) after scrutiny, be retained in the honorable role of being an assumption.

(*b*) The order of presentation of propositions may be so altered that a theorem (proved proposition) in the first order will appear as an assumption (unproved proposition) in the second. The proposition is unchanged, but its role is altered, just as a given scout troop may be given different responsibilities at two successive scout rallies.

(*c*) Whenever the stage of logical organization is reached, all reasoning, in whatever field, takes on the character indicated in geometry: undefined terms and definitions, assumptions and theorems.

A word should be said about formal statements such as appear in the summary.

A formal statement is a *careful* statement, the sort of thing that should be expected of any person in a position of responsibility. Such a person should learn to choose his words.

The words in a geometrical statement are important for the wrong word may affect the proof of the statement in various ways.

We believe that *practice in the construction of careful statements* should be part of the training for citizenship. This has been deliberately stressed in the later chapters on the circle, and such practice may be arranged for in the earlier chapters without difficulty.

CHAPTER II

The Right Triangle

The right triangle is the most frequently used triangle in practical work and it is fundamental later in analytic geometry (including trigonometry) and the calculus. It has been given deliberate prominence throughout the text.

The student should try to recall what he knows about the right triangle before entering the chapter.

The ideas of the chapter are probably known to the student. The exercises are scale-drawing for the most part, and it may be that only a few exercises need to be done. There is no proof work in the chapter.

Detail by Sections

1. Practical work out-of-doors in right measurement should be encouraged, and a variety of simple instruments for this purpose can easily be made by the class. How is a right angle laid off on the ground? How may the width of a stream (which may be staked off on the school grounds!) be determined? How to find the height of a building? etc.

2. Scale drawing and measurement should be used to develop some feeling for rounded off numbers in simple work. A good scheme is to have measurements made by the class with pooled results. Example: Draw on graph paper a right triangle with sides about the right angle of length 3.75 cm. and 2.85 cm. Measure with a protractor and as closely as possible the angle opposite to the larger side.

The answers will show considerable variation about 52° 46′ (the table reading). Such an exercise serves to show the effect upon the result of (i) the size of the figure; (ii) the size of the pencil point; (iii) the accuracy of the protractor; (iv) other personal factors. Also the exercise tends to make the pupil chary of an overabundance of digits in numbers. In the above example the class will probably agree to record the angle to the nearest degree.

CHAPTER III

Triangle Measurement

This work is likely familiar and the number of exercises needed may not be large. A few simple out-of-door exercises may be useful. The faster student should put emphasis on the exercises in Chapter IV. The slower student may do fewer of the theoretical exercises in Chapter IV and more of the map reading.

Detail by Sections

As stated in the text, when a town is shown with an aerodrome, distance and direction are measured *from the aerodrome* in all exercises. It would have been better to have used the radio range centre instead of the aerodrome, but all answers refer to the latter.

The answers were obtained from the original map and are generally satisfactory. It needs to be noted, as stated on the map, that the reduction from the original makes a new distance scale necessary. One inch on the sketch represents 12 miles *not* 8 miles.

There is one error in the sketch; the town of *Waterton Park* should be at (Lat. 49° 03′ N., Long. 113° 55′ W.)

The following answers should be amended:

Ex. 8 to read—

(*a*) N. 45° E., N. 28° W.
(*b*) S. 64° E., S. 56° W.

Ex. 18 (*d*) to read—

(49° 26′ N., 112° 53′ W.)

Attention is drawn to Exercises 10 and 17. Ex. 10 shows the utility of negative angle. Here the extension of the meaning of angle erases the exception in the rule for positive angles. Suppose the magnetic variation is 23° E. If the given direction is less than 23° T. the rule as stated gives a negative magnetic bearing. Thus if the course is in the direction 10° T., the magnetic course is — 13° M. If the use of negatives is excluded, the magnetic course is 347° M. and the rule has to be clumsily amended.

Ex. 17 is another good exercise. The rule (Theorem) as stated when applied to Ex. 16 gives X and Y correctly but not Z, since X, Y and Z are angles less than 180°. The phrase in the theorem "The angle between two directions" is (purposely) vague and should be defined separately.

The use of the beam signals A (dot-dash) and N (dash-dot) will likely be familiar to some radio-minded boy in the class. Note how the signals are sent out in the same direction at each station and that from station to station the signals change over. On the "beam" the signals overlap and the pilot hears a succession of long dashes.

CHAPTER IV.

The Triangle, Side and Angle Properties

It may not be necessary to do many of the exercises of this chapter. Whatever work is done should lay stress on the reasoning used and the style of proof. Chapter X may be used whenever necessary.

Detail by Sections

1. The mode of proof employed here is a useful one, but the student may at first be confused by the reversal involved. Simple non-geometrical examples of this type of reasoning are not difficult to devise. The student should learn to list clearly the given and required elements in each proposition.

The inclusion of a paragraph of discussion in the statement of the proof may help to dispel the atmosphere of artificiality which a written proof sometimes conveys.

The teacher may wish to add to the list of paragraphs in the proof of a proposition, two further paragraphs before (v), namely (*a*) definitions; (*b*) assumptions; which emerge from the discussion.

The use of theorem numbers for reference purposes makes exercise correction simpler. The brief explanatory statement of the theorem should also be given in the reference. The student may well write out in a special note book the theory as it is developed progressively, as given in the summaries at the ends of Chapters I, IV, etc.

2. No comment.

3. The analysis of a good mystery story may interest a group of pupils in the class. The pupils should be encouraged to collect examples of reasoning from everyday situations, and to analyse these.

CHAPTER V.

Constructions*

The length of time to be spent on this chapter will depend upon the understanding of constructions shown by the student.

Constructions require good instruments. There is little joy in constructional work with poor equipment. Some boys may wish to use the engineering constructions to draw some simple plan to large scale. There are many magazines familiar to boys which contain scale plans of all kinds.

Detail by Sections

1. The review of the constructions should concern the *how* rather than the *why*, the method rather than the proof of the method. The method for each construction is detailed in Chapter X. The proofs are suggested as later exercises (Ex. 19, §2). If the proofs give difficulty the suggestions of Chapter X may be consulted.

The use of construction is a likely source of confusion to the pupil. He may feel that the injunction "draw a line through a point ⊥ to a given line" is easily met with the use of a set square. So it is, as a matter of *drawing*. But in a *proof* which involves perpendicularity, the drawing of a perpendicular with a set square may be of little direct help since it may not yield the insight required for the proof.

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*Constructions are shown in italics in the text while theorems are shown in bold face print.

2. The use of collapsing compasses (p. 48). According to the Greeks, when a circle was drawn the compasses folded up, so that the use of dividers was contrary to their game of geometry. The construction of Euclid, p. 51, Ex. 20 is a process by which transfer of distance is made possible without dividers. Further detail concerning the "impossible" problems may be had in any history of mathematics (see reference list).

Practice in the phrasing of careful statements of possible new theorems may be had in the exercises.

3. The making of the models may interest a group of students.

CHAPTER VI

Polygons, Area, the Pythagorean Theorem

The theorems of this chapter are for the most part familiar ones. These theorems are listed at the end of the chapter and hints for their proof are given in Chapter X.

The student may need to spend but little time on the chapter. Later reference as needed may be made to the materials.

There are plentiful proof exercises.

Detail by Sections

2. Ex. 2 (*a*) Such strips are available in toy construction sets.

3. Ex. 17-18 suggest out-of-door work.

4. The method suggested for constructing a right angle is a good exercise for out-of-doors.

Three dimensional models are useful aids, e.g., a balsa wood model for Ex. 12 is simple to construct.*

CHAPTER VII.

Loci

The locus as the path of a moving point is a useful notion, and is continually applied in the processes of analytic geometry and calculus.

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*Balsa wood strips can be obtained, with the necessary liquid cement, from any store which sells model aeroplane kits.

There is another view of the matter which is also important and logically more satisfactory, namely, the locus as totality, as all the points which satisfy the given conditions.

It is this second idea which motivates the geometrical proof of a locus, for a locus implies a proposition and its converse, viz.: (1) If P is equidistant from R and S then it is on the perpendicular bisector of RS. (2) If P is on the perpendicular bisector of RS then P is equidistant from R and S. Both these statements are implied in Th. 26. Compare p. 122. *All* the points are wanted which satisfy the given conditions (the whole truth) and no other points are wanted (nothing but the truth).

We have refrained from phrasing a formal definition of *locus*. The dual aspect of the matter, indicated above, makes a satisfying definition difficult. The pupil will find this a valuable exercise, when the chapter has been worked through. This matter of difficulty of definition appears in the next chapter, and the definition of a locus is referred to there.

Detailed comments on the sections are hardly necessary. Further locus exercises are given in the revision exercises at the end of the book.

CHAPTER VIII

Similar Figures

The treatment differs from the usual one. The subject is never easy; at least in later courses pupils are usually hazy about the basic ideas. It is hoped that the present treatment will make the topic more understandable.

The exercises of the chapter are almost entirely oral and numerical. The proofs of the theorems should be deferred until the facts which they embody have been practised in the exercises.

It is difficult to avoid the implication of this chapter that a figure is a region in the plane or a solid in space, viz.: a triangle is like a piece of cardboard in contrast to a roof truss; a sphere is not so much a child's rubber ball as a solid baseball, etc.

There is no harm in this as long as the student is expected to distinguish between curve and plane segment, surface and solid, whenever the need arises for so doing.

Fig. 8.6 contains a misprint. The number 10 should be the letter *w*.

Detail by Sections

1. No comment.

2. The proofs should be deferred. Simple numerical exercises with triangles used orally, will help to make the facts of the theorems clear.

3. No comment.

4. The use of graph paper is the simplest and most important method of putting figures into correspondence.

5. The meaning of similar figures must be held clearly in mind if the theorem content of this section is to be appreciated. Appreciation is the first necessity. Proofs should be deferred.

6. The chief value of this section is that it serves to shake over-confidence in the deceitful symmetry of Theorems 30-33.

7. Ex. 6 should be noted.

Chapter Summary

Once the notions of similiarity have been grasped, the proofs of the theorems should be considered. Hints are given in Chapter X. The proofs should be written in good style. The use of Theorem 31 as an assumption will challenge some students to try to prove this proposition. This sort of effort is always interesting; the proof will require the use of other assumptions which the student should seek to make explicit.

CHAPTER IX

Simple Trigonometry

The treatment stresses the use of variables, and the definitions are so stated as to be extended directly for any angle.

The letter a for angle has been used in preference to θ, which is the letter commonly used in texts in trigonometry. Since the student has used a for angle in the earlier chapters, it was thought that its continuance would help the student. Should he go on later with trigonometry the introduction of θ at that time should not prove difficult.

The use of tables such as *Knott, Four Figure Tables* is to be preferred to the use of the table in the text. If the text table is used, reading should be made to the nearest half-degree only.

Set A of the revision exercises at the end of the book requires simple trigonometry.

No comment on the chapter sections seems necessary.

CHAPTER X.

The Proof of Theorems

As has been noted earlier, the materials of this chapter are to be used from the beginning of the course as the need arises for them.

Detail by Sections

1. It is important that the pupil should know the main definitions in any subject, but what is more important, he should be sharply aware that a definition is an agreement which has been entered upon by him with other contracting parties. Definitions, like other forms of contract, may be good, bad or indifferent.

To learn that a parallelogram is a quadrilateral with opposite pairs of sides parallel is not enough. The pupil needs to see that this is only one of several possible meanings and that this one has been selected, by experience, as preferable to the others.

A good definition is known only by its fruits. Even dictionary meanings, as a well known logician reminds us, "only offer advice to the user of a word" (F. C. Schiller, *Logic for Use*, p. 62).

The pivotal assumptions should be learned again in a setting of free discussion and good sense.

Many people treat assumptions as if they were unnecessary. We often hear it said "That, after all is only an assumption." The words *after all, only* suggest contempt. The pupil should see that every bit of thinking we do must be based of necessity on assumptions. These assumptions may be explicitly stated, or as is more common, may be implied. Thus the Canadian Constitution was based on the major assumption that the Act of Constitution should avoid what were thought to be the mistaken ideas of the Constitution of the United States. (Muir, *Short History of the British Commonwealth*, Vol. 2). A mere reading of the Act however does not reveal this basic "principle" or "purpose."

The pupil should find examples from his own experience and reading. The exercises given are merely suggestive.

It is hoped that the student will begin to appreciate the logicial organization of propositions, how each depends upon earlier ones.

The symbol of a chain, viz.: chain of reasoning, is usually too simple a picture. The shrub or tree is better. A branch of one shrub may be grafted upon another. A shoot of the old tree may be planted to produce a new tree. What was branch in the old structure may become stem in the new. This also can be seen in genealogies; an obscure prince marries into the Royal House; a new House stems off the old, whereas the old lapses and finally dies out.

3. The teacher should realize that the *converse* is only one possible type of derivative theorem. Other important types are discussed in *Lazar* (See reference list.)

Part II.—Hints for the Proof of Theorems

These should be consulted only when the student has made a serious effort of his own to formulate a proof.

One of the main functions of proof in any subject is to foster a growing confidence in the worker's ability to organize the materials into some semblance of coherence. A proof concerns the relation of one proposition to other propositions.

Any intelligent approach to the teaching of geometry will have as one of its bases this nurture of pupil-confidence in the pupil's own organizing ability.

CHAPTER XI.

The Circle, Length, Area, Chords

The succeeding chapters require the exploratory spirit on the part of pupil and teacher. It is true that the facts discovered in a given chapter are listed formally as propositions at the end of the chapter, mainly as a check on the class-work. The student, however, should be encouraged to seek his facts and formulate his theorems and to refrain from consulting the theorem list at the end of the chapter until such effort has been made.

The exercises of the first eight sections are wholly numerical; exercises requiring the use of trigonometric functions are included, but segregated.

The proofs of Theorems 40, 41 and corollaries should be written out in detail by the student.

CHAPTER XII.

The Circle, Tangents

The proper definition of a tangent to a curve is necessary to the calculus. The ground has been prepared for this definition by locus thinking and it is not difficult to appreciate. The tangent to a circle is treated as a special case of the tangent to any curve.

Here as in Chapter XI and in the subsequent chapters the stress is on class exploration and organization. The facts, as usual, are incorporated in formal statements at the end of the chapter.

The slow pupil may well be excused the exercises involving constructions and proofs.

CHAPTER XIII.

The Circle, Angle Properties

The organization of this chapter follow the lines already indicated for Chapters XI and XII. It was thought advisable to begin with the semicircle, for the property described occurs often in later applications and the proofs of Theorems 46 and 47 are somewhat easier than those of Theorems 48 and 51.

As in the two previous chapters numerical exercises are given to enable the student to learn the properties themselves. Section 6 may well be omitted for all but a very few students.

In Ex. 1, p. 184, **D** and **E** should be interchanged throughout the statement of the exercise. On p. 196 Th. 49 should precede Th. 48. The proof of Th. 48 depends upon Th. 49. The student should remember these theorems.

CHAPTER XIV.

Space Figures

This chapter will serve as a useful reference chapter during the working of three dimensional exercises in the earlier chapters.

The materials of this chapter lend themselves to practical work, particularly model making. Cardboard, balsa wood and wire will suffice. Thus the cuboid with its diagonals may be made of balsa wood strips, so with the parallelepiped, etc., to say nothing of composite figures, such as is suggested by the dormer window. Modelling exercises are also suggested in sections **9** and **10.**

In addition, interesting work may be done with volumes, from the standpoint of cubic capacity. Thus the content of a can for variable height may be had by filling up with water to the desired level, then pouring out into a calibrated beaker. Sphere volume can be had by displacement; drop heavy spheres into water in a calibrated beaker, etc.

Whatever of this practical work is desired may be done during the work of the earlier chapters.

Surfaces which are (i) "developable;" (ii) "non-developable" on the plane may easily be illustrated with (i) a paper cone and cylinder; (ii) a baseball cover.

Similar figures are often used to depict statistics. These furnish good illustrations of the areas and volumes of similar figures. A collection of crystals should be visited, or assembled. A recent small book, *The Nature of Crystals*, A. G. Ward (Blackie 3/6), may prove of much interest to some pupils.

REFERENCES

The Mathematical Association of Great Britain has published two reports on the teaching of geometry:

The Teaching of Geometry in Schools (2nd Ed., 1925, Bell and Sons, London).

A Second Report on the Teaching of Geometry in Schools (1938, Bell, London).

The journal of the Association is:

The Mathematical Gazette (Bell, London).

The U.S. National Council of Teachers of Mathematics publishes a year book. The following are of special interest here:

The Teaching of Geometry (1930, Columbia Univ. Press, New York).

The Nature of Proof (1938, Columbia Univ. Press, New York).

The journal of the Council is:

The Mathematics Teacher (525 W. 120 St., New York).

The following books may be of assistance:

The Importance of Certain Concepts and Laws of Logic for the Study and Teaching of Geometry, Lazar (1938, Banta Publishing Co., Menasha, Wis.).

Field Work in Mathematics, Shuster and Bedford (1935, American Book Co., New York).

The Elements of Euclid (Home Univ. Lib.), (1933, Dent, London).

A New Geometry, Siddons and Snell (1938, Cambridge).

A School Geometry, Forder (1930, Cambridge).

Outline of the History of Mathematics, Archibald (4th Ed., 1938, Math. Assoc. of America, Oberlin, Ohio).

A Short History of Mathematics, Sanford (1930, Houghton Mifflin, Boston).

Manual of Greek Mathematics, Heath (Oxford). The *Manual* is an abridgment of Sir T. L. Heath's two volume history of Greek Mathematics.

Fundamental Concepts in Algebra and Geometry, Young (Macmillan).

Thinking to Some Purpose, Stebbing (Penguin Books Ltd., London). This book has many illustrations of common difficulties in everyday thinking.

APPENDIX A.

Significant Figures

In computation or in measurement more digits may be temporarily recorded than are needed or justifiable. In such cases the number is *rounded-off* according to some rule to the proper number of digits. The usual rule is: *When the first digit of those to be dropped is any one of* **0, 1, 2, 3, 4,** *no change is made in the last digit retained; otherwise that digit is increased by* **1.**

Thus approximations in rounding-off 585.093 are:

585.09, 585.1, 585, 590, 600.

Note that the last two numbers are given to the nearest 10 and the nearest 100 respectively. The number 600, without further explanation, may of course mean 6×10^2, 6.0×10^2 or 6.00×10^2.

The *significant figures* of a number in decimal form are those digits which still have meaning when the decimal point is erased. Thus the digits which make sense after erasing the decimal point in the numbers

8.9020, 89.020, 0.0089020

are the five digits, 8, 9, 0, 2, 0. We speak of each of these preceding three numbers as given to five (significant) figures.

The number of significant figures in 600 (see above) is not made apparent by the definition just given. It is necessary in such a case to indicate the significant zeros in some special way. A simple schedule is to underline the zeros that are signficant, viz.: 600, $6\underline{0}0$, $6\underline{00}$.

The meanings of these numbers should be noted. Thus 600 means that the number lies between 550 and 650, whereas $6\underline{0}0$ means that the number lies between 595 and 605 and the number represented by $6\underline{00}$ lies between 599.5 and 600.5.

It will be noted that there is no connection between the number of decimal places and the number of significant figures. The roles of places and figures in the combination of rounded-off numbers are quite distinct.

When the length x in. of a rod is stated to be 2.37 in., we may assume that x lies between 2.365 and 2.375, so that the rounded-off number 2.37 is in doubt by as much as ± 0.005. We may speak of ± 0.005 as the *possible error* in the number with a corresponding *possible percentage error* of ± 0.2%.

The possible errors of the numbers 432, 4.32, 0.0432, etc., are all different, namely, ± 05, ± 0.005, ± 0.000,05, but their percentage errors are the same, about ± 0.12%. Likewise these numbers are given to different places of decimals, but have the same number (three) of significant figures.

It is therefore not difficult to see that—

(1) The greater the number of significant figures, the smaller the percentage error in the number.

(2) Numbers with the same number of significant figures have their percentage errors lying within a definite range, e.g., the percentage error for 3 figure numbers lie between 0.5 and 0.05 per cent.

Since the percentage error in a product can be shown to be the sum of the percentage error in the factors, the factor with the least number of significant figures really determines the percentage error of the product. The product of 4.83 and 608.37 cannot have more than three significant figures, since 4.83 is given to three figures.

So for a quotient; the result of dividing 82.76 by 5.3 should be given only to two figures.

How many figures should be retained in sine 46° 32′? The working principle which is satisfactory for the computation of any of the elementary functions is to carry as many figures as are given in the independent variable. Thus 46° 32′ is in reality given to four figures and hence the sine of this angle should be carried to four figures. We note that this principle holds for the *mantissa* of a logarithm, i.e., if the number is given to five figures, so should the mantissa be carried to five figures.

Decimal places pertain to addition and subtraction and the number with the fewest decimal places controls the accuracy of these operations.

As far as the student is concerned he should be helped to be conservative in the number of significant figures used in computation. He should never carry more than four figures in this course. As a rule the result of his calculation should show one figure less than the number of figures used in the computation.

APPENDIX B.

Drawing Instruments

Construction to scale to be satisfactory requires reasonably good drawing instruments. These are not always available, but with a little care and ingenuity these can be made by the student.

(*a*) *Decimal Inch Scale—*

The scale should be made on flexible tough cardboard, six to ten inches in length. First of all, scale off in inches. Each inch should then be subdivided in fifths; use similar triangles to obtain fifths accurately by reduction. This reduction can be made on paper and then carefully transferred to the cardboard scale. If desired, tenths may also be shown.

Measurement with the decimal inch scale gives results in decimal form directly and avoids the awkward conversion usual with the ordinary scale. Moreover, as a recent writer (Paul Hoffman, *Compressed Air Magazine,* February, 1940), puts it, "Draftsmen, pattern makers, mechanics and engineers will bless the day" when the decimal scale displaces the inch scale now in use.

(*b*) *The Protractor—*

The protractor is first made on graph paper and then gummed on a rectangular sheet of tough cardboard. The essential thing is to get the angles made accurately; this is best done with a table of sines.

If the graph paper is scaled to centimetres, use 10 cm. as the unit and draw a semicircle, radius 10 cm., on the graph paper. If inch paper is used, take 5 in. as radius.

From a table of sines construct angles at the centre of the semicircle at 5° intervals from 0° to 180°. Thus sine 10° = 0.1736; find the points of the circumference of the semicircle which are $10 \times 0.1736 = 1.736$ cm. from the base of the semicircle (there are two points corresponding to angles of 10° and 170°). Draw the arms of the angles to within 2 cm. of the centre. Each 5° arc may now be subdivided by inspection into degree intervals, and the lines drawn lightly, towards the centre.

The graph sheet is then cut to size, say 8 in. × 4 in., with the centre of the semicircle as the mid point of the 8 in. base of the rectangle. This may then be labelled properly and gummed on a corresponding rectangle of tough cardboard.

The tangent function may be used to construct a protractor on one arm of the right angle of the 45° — 45° — 90° set square. This protractor will give angles from 0° to 45°, and is therefore sufficient for the construction of an angle of any size. A decimal inch scale can be made on one of the remaining sides of the set square.

(c) *Set Squares*—

These may be of wood; the 45° square can have sides 8 in. × 8 in., and the 30° — 60° square, hypotenuse 10 in. and side 5 in. The triangles should be carefully drawn on the wood, cut out with a fine saw with the edges lightly sanded. They should be tested for trueness.

(d) *Dividers and Compasses*—

The best available, reasonably priced instruments should be obtained. These should be kept in good trim. The pencil should be Grade H, if possible.

The student should be encouraged to work on a plane board as a drawing board. T-squares are useful. Grade H pencil should be used if possible. Drawings should be accompanied with clear descriptive statements and adequate reference and title; work need not be inked over except for display purposes.

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